Statistical Fluctuations of Electromagnetic Transition Intensities in pf-Shell Nuclei

A. Hamoudi[1], R.G. Nazmitdinov[1] and Y. Alhassid[2]

[1] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

[2] *Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520, USA*

(February 9, 2008)

Abstract

We study the fluctuation properties of $E2$ and $M1$ transition intensities among $T = 0, 1$ states of $A = 60$ nuclei in the framework of the interacting shell model, using a realistic effective interaction for pf-shell nuclei with a ^{56}Ni core. It is found that the $B(E2)$ distributions are well described by the Gaussian orthogonal ensemble of random matrices (Porter-Thomas distribution) independently of the isobaric quantum number T_z. However, the statistics of the $B(M1)$ transitions is sensitive to T_z: $T_z = 1$ nuclei exhibit a Porter-Thomas distribution, while a significant deviation from the GOE statistics is observed for self-conjugate nuclei ($T_z = 0$).

PACS numbers: 24.60.-k, 24.60.Lz, 21.60.Cs, 21.10.Ky

Random matrix theory (RMT) [1] was originally introduced to explain the statistical fluctuations of neutron resonances observed in compound nuclei [2]. The theory assumes that the nuclear Hamiltonian belongs to an ensemble of random matrices that are consistent with the fundamental symmetries of the system. In particular, since the nuclear interaction preserves time-reversal symmetry, the relevant ensemble is the Gaussian orthogonal ensemble (GOE). Bohigas *et al* [3] conjectured that RMT describes the statistical fluctuations of a quantum system whose associated classical dynamics is chaotic. More recently, a proof for this conjecture was proposed by mapping the problem on the supersymmetric non-linear sigma model [4]. RMT has become a universal tool for analyzing statistical fluctuations in chaotic systems [5,6].

The chaotic nature of the single-particle dynamics in the nucleus can be studied in the framework of the mean-field approximation. The interplay between shell structure and fluctuations in the single-particle spectrum can be understood in terms of the classical dynamics of the nucleons in the corresponding deformed mean-field potential [7,8]. However, the residual nuclear interaction mixes different mean-field configurations and plays a crucial role in determining the nature of the statistical fluctuations of the many-particle spectrum and wavefunctions. The statistics of the low-lying collective part of the nuclear spectrum has been studied in the framework of the interacting boson model, in which the nuclear fermionic space is mapped onto a much smaller space of bosonic degrees of freedom [9,10]. Because of the relatively small number of degrees of freedom in this model, it was also possible to relate the statistics to the underlying nature of the mean-field collective dynamics. Since at higher excitations additional degrees of freedom (such as broken pairs) become important [11], it is necessary to analyze the effect of interactions on the statistics in much larger model spaces. The interacting shell model offers an attractive framework for such studies where realistic effective interactions are available (in finite model spaces), and the basis states are labeled by exact quantum numbers of angular momentum, isospin and parity [12].

RMT makes definite predictions for the statistical fluctuations of the eigenfunctions as well as the spectrum. The electromagnetic transition intensities in a nucleus are observables

that are sensitive to the wavefunctions, and the investigation of their statistical distributions should complement the more common spectral analysis [9,10]. $B(M1)$ and $B(E2)$ transitions were recently analyzed in ^{22}Na [13] in the framework of the shell model and found to follow the Porter-Thomas distribution [14], in agreement with RMT and consistent with the previous finding of a Gaussian distribution for the eigenvector components [15]- [18]. In addition, the distributions of $E2$ and $M1$ matrix elements did not show any sensitivity to spin and isospin [13].

Most studies of statistical fluctuations in the shell model have been restricted to lighter nuclei ($A \lesssim 40$) where complete $0\hbar\omega$ calculations are feasible (e.g. sd-shell nuclei). It is of interest to investigate how the statistics evolves with increasing mass number. In the present paper we study the fluctuation properties of electromagnetic transition intensities in nuclei with $A \sim 60$. We find that the $B(E2)$ distributions are Porter-Thomas, but that the $M1$ statistics is sensitive to isospin. In particular the $B(M1)$ distributions in self-conjugate nuclei show a significant deviation from Porter-Thomas. The calculations are performed in the pf shell with ^{56}Ni as a core, i.e. we assume a fully occupied $f_{7/2}$ orbit and consider all possible many-nucleon configurations defined by the $0f_{5/2}$, $1p_{3/2}$ and $1p_{1/2}$ orbitals. The effective interaction is chosen to be the isospin-conserving F5P interaction [19]. This interaction is successful in describing the mass range $A \sim 57-68$. The calculations were performed with the shell model program OXBASH [20].

Denoting by $B(\varpi L; i \to f)$ the reduced transition probability from an initial state $|i\rangle$ to a final state $|f\rangle$, with $\bar{\omega}$ indicating the electric (E) or magnetic (M) character of the transition, and 2^L the multipolarity, we have [21]

$$B(\varpi L; J_i T_i T_z \to J_f T_f T_z) = \frac{|\delta_{T_i T_f} M_{is}(\varpi L) - (T_i T_z 10|T_f T_z) M_{iv}(\varpi L)|^2}{(2J_i+1)(2T_i+1)} \, . \tag{1}$$

Here $M_{is}(\varpi L)$ and $M_{iv}(\varpi L)$ are the triply reduced matrix elements for the isoscalar and isovector components of the transition operator, respectively. Note that these matrix elements depend on J_i, T_i and J_f, T_f but not on T_z. For $\Delta T = 0$ transitions ($T_i = T_f = T$) the isospin Clebsch-Gordan coefficient in Eq. (1) is simply given by

$$(TT_z10|TT_z) = T_z/\sqrt{T(T+1)}\ . \tag{2}$$

It follows that the isovector component in Eq. (1) is absent for self-conjugate (i.e. $T_z = 0$) nuclei. Thus the statistics of the isoscalar component of an electromagnetic transition operator can be inferred directly from $T_z = 0$ nuclei. Consequently, we can test the sensitivity of the statistics to the isovector and the isoscalar contributions.

To study the fluctuation properties of the transition rates, it is necessary to divide out any secular variation of the average strength function versus the initial and final energies. We calculate an average transition strength at an initial energy E and final energy E' from [22,10]

$$\langle B(E,E')\rangle = \frac{\sum_{i,f} B(\varpi L; i \to f) e^{-(E-E_i)^2/2\gamma^2} e^{-(E'-E_f)^2/2\gamma^2}}{\sum_{i,f} e^{-(E-E_i)^2/2\gamma^2} e^{-(E'-E_f)^2/2\gamma^2}}\ . \tag{3}$$

For fixed values of the initial (J_i^π, T) and final (J_f^π, T) spin, isospin and parity, we calculate from Eq. (1) the intensities $B(\varpi L; i \to f)$. All transitions of a given operator (e.g. $M1$ or $E2$) between the initial and final states of the given spin, parity and isospin classes have been included in the statistics. We remark that the energy levels used in (3) are the unfolded energy levels [23], characterized by a constant mean spacing. The value of γ has been chosen to be large enough to minimize effects arising from the local fluctuations in the transition strength but not so large as to wash away the secular energy variation of the average intensity. In the present calculations we used $\gamma = 4$. We renormalized the actual intensities by dividing out their smooth part

$$y_{fi} = \frac{B(\varpi L; J_i T T_z \to J_f T T_z)}{\langle B(E,E'\rangle}\ , \tag{4}$$

and histogrammed them into bins equally spaced in $\log y$. In RMT there is a large number of weak transitions, and we use a logarithmic scale to display small values of y over several orders of magnitude. For each transition operator and classes of initial and final states we fit to the calculated distribution a χ^2 distribution in ν degrees of freedom [24]

$$P_\nu(y) = (\nu/2 < y >)^{\nu/2} y^{\nu/2-1} e^{-\nu y/2<y>}/\Gamma(\nu/2)\ . \tag{5}$$

For $\nu = 1$ this distribution reduces to the Porter-Thomas distribution. In systems with mixed classical dynamics, it is found that ν decreases monotonically from 1 as the system makes a transition from chaotic to regular motion [22].

We first examine all $E2$ and $M1$ $2^+, T \to 2^+, T$ transitions ($T = 0$ or $T = 1$) in $A = 60$ nuclei. $T = 0$ states exist only in $T_z = 0$ nuclei, i.e. ^{60}Zn in our case. However, the $T = 1$ states form isobaric multiplets in ^{60}Co, ^{60}Zn and ^{60}Cu. In the latter case we studied the statistics in both $T_z = 0$ and $T_z = 1$ nuclei. Because of the vanishing of the isovector Clebsch-Gordan coefficient in Eq. (1), the transitions in ^{60}Zn ($T_z = 0$) are purely isoscalar. For each transition operator we sampled $56^2 = 3136$ and $66^2 = 4356$ intensities for $T_z = 0$ and $T_z = 1$, respectively. The calculated distributions (histograms) of the $B(E2)$ (left panels) and $B(M1)$ (right panels) $2^+, T \to 2^+, T$ transitions are shown in Fig. 1, and compared with the Porter-Thomas distribution (Eq. (5) with $\nu = 1$).

The distributions of transitions within the $T = 1$ states are shown in the top ($T_z = 1$) and middle ($T_z = 0$) panels. Since our interaction is isospin invariant, the energy levels of an isobaric multiplet are degenerate, and the spectral statistics of $T = 1$ states must be the same in both $T_z = 0$ and $T_z = 1$ nuclei. We find that the level spacing and Δ_3 statistics are in agreement with GOE. However, while the $B(E2)$ distributions are all in agreement with the GOE prediction, the $M1$ distributions show strong sensitivity to T_z. For $T_z = 1$ the distribution of the $M1$ intensities is Porter-Thomas, but in self-conjugate nuclei we find that the $M1$ distribution deviates significantly from the GOE limit. The dashed line in Fig. 1d shows a χ^2 distribution (5) with $\nu = 0.64$. Although it does not fit well the calculated distribution, the smaller ν is consistent with the observed larger number of weak transitions as compared with a Porter-Thomas distribution. A similar deviation is observed in the $M1$ distribution for the $2^+, T = 0 \to 2^+, T = 0$ transitions (see Fig. 1f), while the $B(E2)$ probabilities are distributed according to Porter-Thomas (Fig. 1e). The deviation from the GOE occurs for the $M1$ transitions in self-conjugate nuclei (e.g. ^{60}Zn), where the matrix elements are purely isoscalar. In $T_z = 1$ nuclei both isoscalar and isovector components contribute to the $M1$ transitions. However, since the isoscalar $M1$

matrix elements are much weaker than the isovector M1 [21], the latter dominate and the distributions are restored to their GOE form.

We also investigated the statistics of the $0^+, T = 1 \to 1^+, T = 1$ transitions and its dependence on the number of final states. In the top panels of Fig. 2 we compare the $M1$ distribution ($T_z = 1$) between all 16 $0^+, T = 1$ initial states and the lowest 16 $1^+, T = 1$ final states with the corresponding distribution when all 54 final states are taken into account. Both seem to give similar distributions, but the statistics is poorer in the first case.

The $M1$ strength function is composed of a spin contribution and an orbital contribution, each dominating in a different energy region. We find that the orbital part is in close agreement with the Porter-Thomas statistics while the dominating spin part shows a slight deviation (see Fig. 2c,d).

Finally, we tested the sensitivity of the statistics to the initial and final spin. In Fig. 3 we show the $B(E2)$ distributions for the $2^+ \to 4^+$ and $4^+ \to 4^+$ transitions between states with $T = 0$. The $B(E2)$ distribution for the $2^+ \to 4^+$ transitions is well described by Porter-Thomas, as in the $2^+ \to 2^+$ case of Fig. 1e (in both cases $J_i = 2^+$). However, the $4^+ \to 4^+$ transitions do show a small deviation from the Porter-Thomas limit. This suggests that the class of 4^+ states is slightly less chaotic.

In conclusion, we have studied the distributions of $B(M1)$ and of $B(E2)$ transition strengths in pf-shell nuclei with $A \sim 60$. While most of the calculated distributions are in close agreement with the Porter-Thomas distribution predicted by the GOE, we find that the $M1$ transitions in self-conjugate nuclei ($T_z = 0$) deviate significantly from the RMT limit. For these nuclei the $M1$ transitions are purely isoscalar and relatively weak compared with transitions in $T_z = 1$ nuclei, which are dominated by the isovector component of $M1$.

The authors are grateful for useful discussions with F. Dittes. We thank V. Ponomarev for assistance during the preparation of the manuscript. This work was supported in part by the Department of Energy grant No. DE-FG-0291-ER-40608.

REFERENCES

[1] M.L. Mehta, *Random Matrices* 2nd ed. (Academic Press, New York,1991).

[2] C.E. Porter, *Statistical Theories of Spectra: Fluctuations* (Academic Press, New York, 1965).

[3] O. Bohigas, M.J. Giannoni and C. Schmit, Phys. Rev. Lett. **52**, 1 (1984).

[4] A.V. Andreev, O. Agam, B.D. Simons and B.L. Altshuler, Phys. Rev. Lett. **76**, 3947 (1996).

[5] T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981).

[6] T. Guhr, A. Müller-Groeling and H.A. Weidenmüller, Phys. Reports, (1998) in press.

[7] R.Arvieu, F. Brut, J. Carbonell, J. Touchard, Phys. Rev. A **35**, 2389 (1987).

[8] W.D. Heiss, R.G. Nazmitdinov and S. Radu, Phys. Rev. Lett. **72**, 2351 (1994); Phys. Rev.C **52**, R1179 (1995); Phys. Rev. C **52**, 3032 (1995).

[9] Y. Alhassid, A. Novoselsky and N. Whelan, Phys. Rev. Lett. **65**, 2971 (1990); Y. Alhassid and N. Whelan, Phys. Rev. C **43**, 2637 (1991).

[10] Y. Alhassid and A. Novoselsky, Phys. Rev. C **45**, 1677 (1992).

[11] Y. Alhassid and D. Vretenar, Phys. Rev. C **46**, 1334 (1992).

[12] V. Zelevinsky, B.A. Brown, N. Frazier and M. Horoi, Phys. Reports **276**, 87 (1996).

[13] A.A. Adams, G.E. Mitchell, W.E. Oemand and J.F. Shriner Jr., Phys. Lett. B **392**, 1 (1997).

[14] C.E. Porter and R.G. Thomas, Phys.Rev. **104**, 483 (1956).

[15] R.R. Whitehead, A. Watt, D. Kelvin and A. Conkie, Phys. Lett. B **76**, 149 (1978).

[16] J.J.M. Verbaarschot and P.J. Brussaard, Phys. Lett. B **87**, 155 (1979).

[17] B.A. Brown and G.F. Bertsch, Phys.Lett. B **148**, 5 (1984).

[18] H. Dias, M.S. Hussein, N.A. de Oliveira and B.H. Wildenthal, J. Phys. G **15**, L79 (1989).

[19] P.W.M. Glaudemans, P.J. Brussaard and R.H. Wildenthal, Nucl.Phys. A **102**, 593 (1976).

[20] B.A. Brown, A. Etchegoyen and W.D.M Rae, the OXBASH shell model code, MSUCL Report No. 524 (1988).

[21] P.J. Brussaard and P.W.M. Glaudemans, *Shell– model applications in nuclear spectroscopy* (North Holland, Amsterdam,1977).

[22] Y. Alhassid and M. Feingold, Phys. Rev. A **39**, 374 (1989).

[23] O. Bohigas and M.J. Giannoni, in *Mathematical and Computational Methods in Nuclear Physics*, edited by J.S. Dehesa, J.M. Gomes and A. Polls (Springer Verlag, New York, 1983).

[24] Y. Alhassid and R.D. Levine, Phys. Rev. Lett. **57**, 2879 (1986).

FIGURES

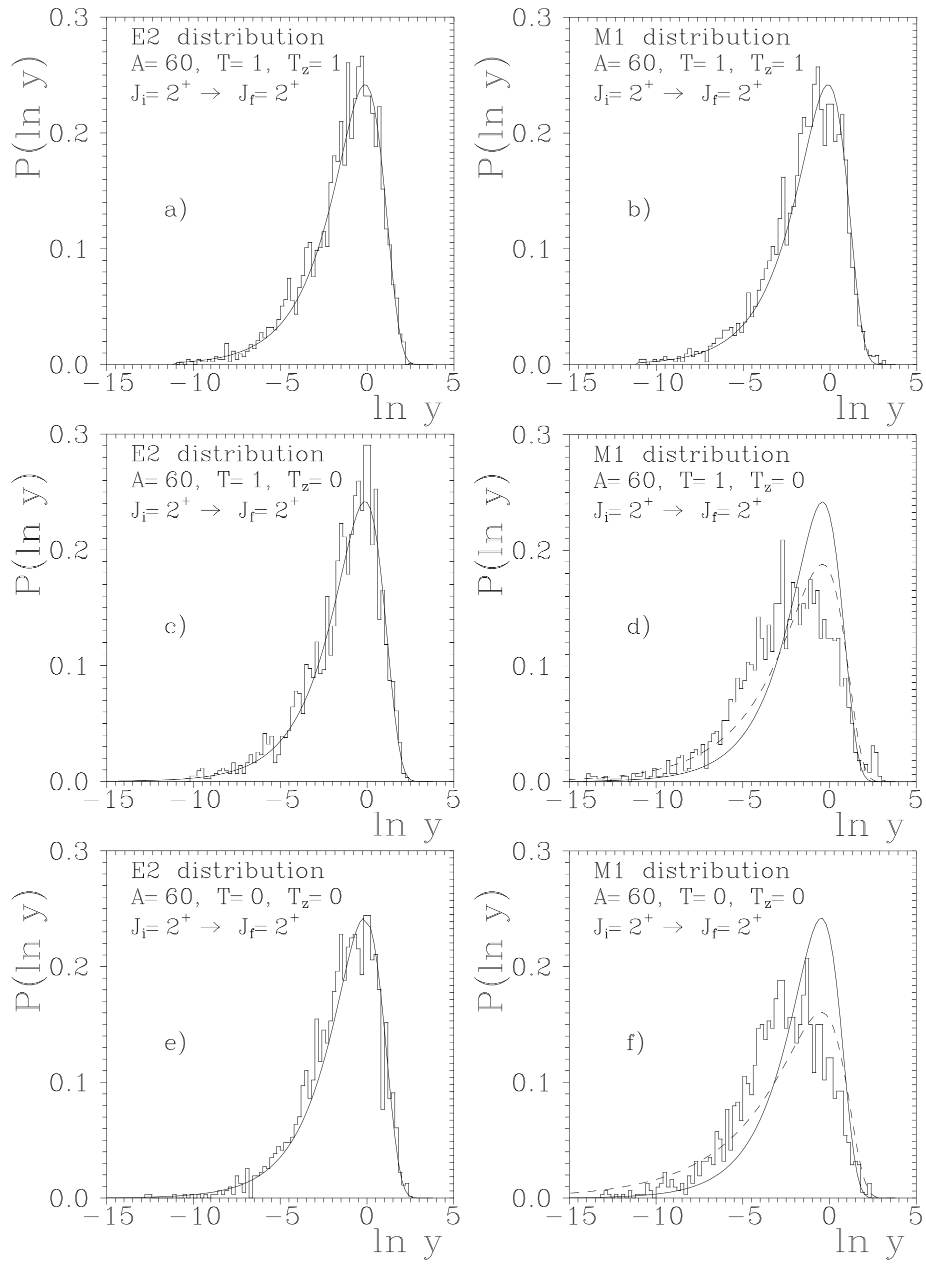

FIG. 1. The $B(E2)$ and $B(M1)$ intensity distributions (histograms) for the $2^+, T \to 2^+, T$ transitions in $A = 60$ nuclei: (a,b) $2^+, 1 \to 2^+, 1$ transitions in ^{60}Co ($T_z = 1$); (c,d) $2^+, 1 \to 2^+, 1$ transitions in ^{60}Zn ($T_z = 0$); (e,f) $2^+, 0 \to 2^+, 0$ transitions in ^{60}Zn ($T_z = 0$). The solid lines describe the Porter-Thomas distribution (Eq. (5) with $\nu = 1$). The dashed lines in d) and f) are Eq. (5) with $\nu = 0.64$ and $\nu = 0.54$, respectively .

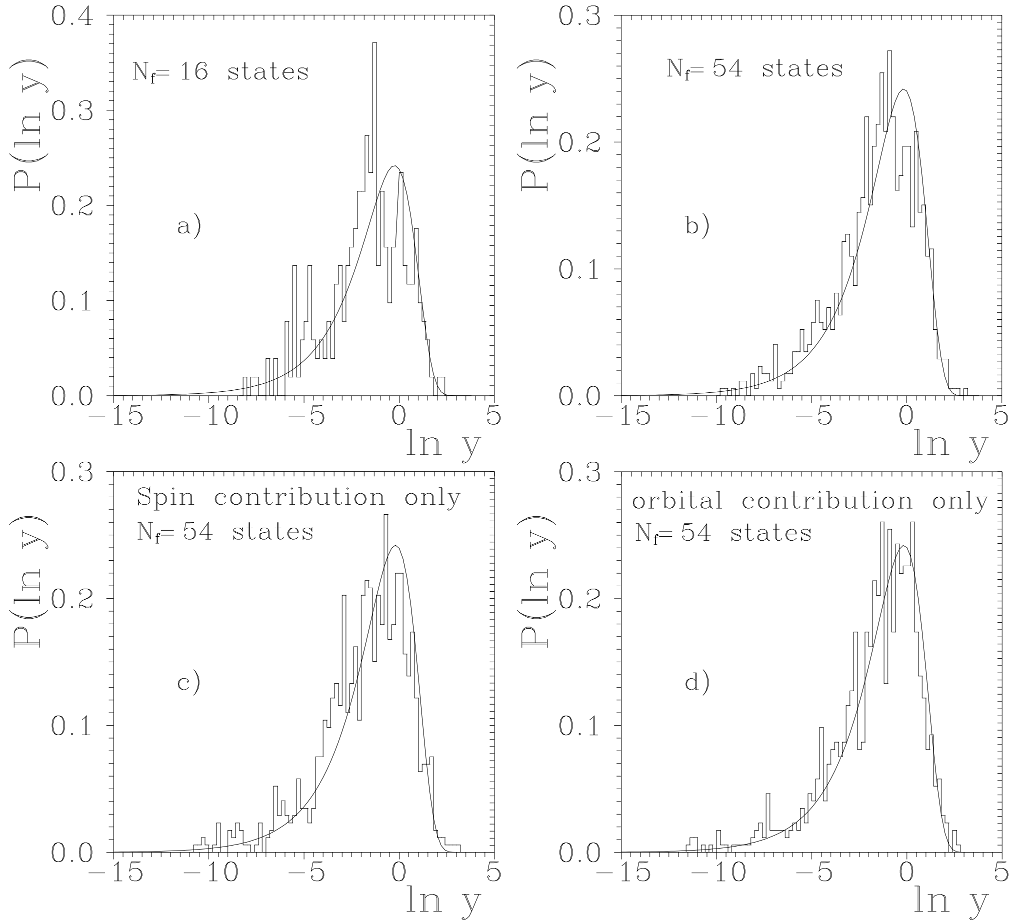

FIG. 2. a,b) Distributions of $M1$ intensities for the $0^+, 1 \to 1^+, 1$ transitions in ^{60}Co ($T_z = 1$) with different number of final states included in the statistics (see text). c,d) The distributions of the spin and orbital contributions to $M1$, for the same transitions as in (b). The solid lines are the Porter-Thomas distributions.

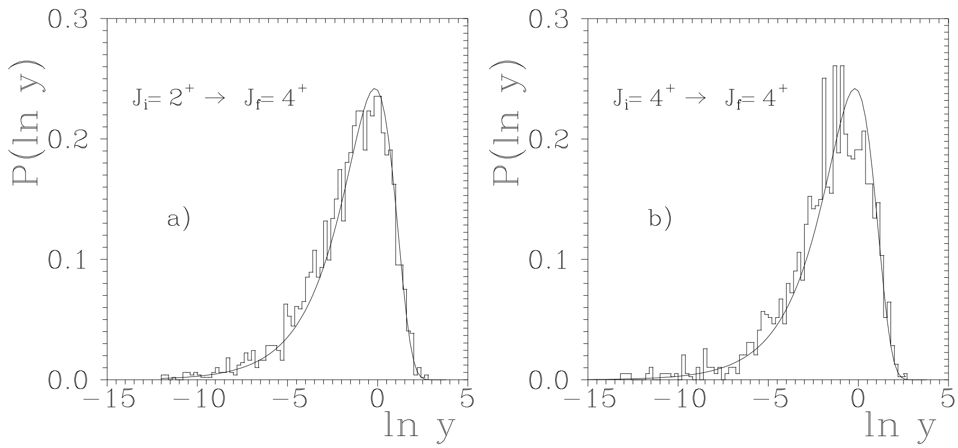

FIG. 3. The $B(E2)$ intensity distributions for the transitions: a) $2^+, T = 0 \rightarrow 4^+, T = 0$; b) $4^+, T = 0 \rightarrow 4^+, T = 0$ in ^{60}Zn.